INVESTIGATIONS MATH PROGRAM

1. **What is Investigations?**

   Investigations in Number, Data, and Space is a complete mathematics program, designed to embody the vision of the rigorous national standards for mathematics developed by the National Council of Teachers of Mathematics (NCTM). It was developed in part with the National Science Foundation and TERC (Technical Education Research Center). *Investigations* was carefully designed around key ideas to invite all K-5 students into mathematics, and was extensively field-tested in a wide variety of classrooms. An extensive body of research on how students learn mathematics informed the curriculum's development. Researchers in several content areas collaborated directly with the development of the curriculum. These included: Douglas Clements and Michael Battista (geometry); Susan Jo Russell, Jan Mokros, Cliff Konold, and Andee Rubin (data); Ricardo Nemirovsky, Cornelia Tierney, and Tracy Noble (mathematics of change). In addition, the developers drew on the large body of educational research carried out over the past 20 years on students' understanding of number and operations.

   The goal of *Investigations* is to develop computational fluency by which a student can develop a deep understanding of math and demonstrate flexibility in the methods they employ. Further, they are asked to explain their methods and produce accurate answers. Typically the number of problems presented are fewer than that of traditional instruction to give students more time to reason, build and test theories, try multiple solutions, and make connections. Presentation of their outcomes is done both orally and in writing giving students the chance to organize their thinking and open a dialogue with their teacher and peers. Additional information can be found on their website: http://investigations.terc.edu/

   Investigations was chosen as the core resource (by teachers and administrators) to support the Lower Merion School District math curriculum in 2009 and meets and often exceeds the Pennsylvania State Standards. In addition, *Investigations* is a coherent and focused K-5 mathematics program that implements the philosophy and content described by the Common Core State Standards (CCSS). The thoughtful implementation of the program began in 2009 with only two modules (Measurement and Geometry) being implemented in grades K-5. In 2010, K-2 implemented the program fully with grades 3 to 5 adding the data module to the existing two modules. In 2011, grades 3 to 5 implemented the program in its entirety with the addition of the number units.

2. **How well does *Investigations* align with the CCSS's Standards for Mathematical Practice?**

   The Standards for Mathematical Practice are closely aligned with the goals and principles that guided the development of every *Investigations* Session. The Practices – which include making sense of and reasoning about mathematics, constructing arguments and explanations, and selecting appropriate tools to model mathematical concepts and solve problems – are and always have been deeply embedded in the fabric of the *Investigations* curriculum and facilitate the teaching and learning of mathematics. Virtually every lesson in *Investigations* includes one or more of these practices.

3. **How well does *Investigations* align with the CCSS Mathematics Content Standards?**

   Investigations in Number, Data and Space can be used to implement the philosophy and content described by the Common Core State Standards (CCSS). Each curriculum unit provides an in-depth study of a specific and related set of mathematical concepts and skills. The design of the materials offers students extended opportunities to make sense of, practice, and develop fluency with the key concepts and skills within a grade level and across grade levels. Full alignment can be achieved by teaching the program fully, as written, and integrating the new content included in *Investigations and the Common Core State Standards*.

4. **What specific data/ research is available on the proven efficacy of Investigations?**
   Research on the effects of *Investigations in Number, Data, and Space* is based on a variety of measures of student achievement and learning, including state-mandated standardized tests, research-based interview protocols, items from research studies published in peer-reviewed journals, and specially constructed paper-and-pencil tests.

   This body of research includes classroom studies, large-scale comparisons across schools, and small-scale comparisons between classrooms. It also includes validation studies that examine impact in school systems that have undertaken a consistent, long-term implementation of *Investigations* as the core mathematics curriculum.

   In an efficacy study conducted by Gatti Evaluation Inc., students randomly assigned to teachers implementing *Investigations* for the first time performed as well or better on the GMADE than their peers, who were randomly assigned to teachers using well-established curricular materials that they were more familiar with. Analyses of sub-test scores show a pattern of results that are especially compelling in the older (4th to 5th grade) cohort. Strong results for the older cohort were also found on a researcher-created performance measure designed to reflect NCTM's Focal Points and assess more complex multi-step problem solving. Analyses of students' self-reported attitudes towards mathematics and mathematics instruction also showed more positive results for the students assigned to *Investigations* classrooms than those in the comparison classrooms.

   During the field-testing of the 2nd edition, an evaluation study was conducted by Kehle and colleagues at Indiana University. In a carefully matched comparison study, within two geographically distinct sites, students in classrooms using the pre-publication versions of the *Investigations* materials performed as well as or better on both standardized tests and researcher-created tests of student achievement. One site showed strong effects in favor of the *Investigations* classrooms while the other site showed no difference between groups.

5. **How is the effectiveness of the program being monitored and assessed in the schools?**
   Administered in September, the K-2 **district made skills and concept inventory** is a focused assessment that is designed to gauge mastery levels of essential learnings (standards) of the previous instructional year. Data collected gives teachers an idea of the entry point into the concepts for each student so they can plan instruction accordingly. The information collected from this assessment can be used to plan differentiated learning opportunities and to provide initial focus of daily routines/learning centers. The information is also used in the curriculum refinement process. (Grades 3 to 5 Inventory assessments will be administered in 2012-13).

For each unit of study, **end-of-unit assessments** from Investigations are used as tools to gauge student learning progress. These assessments, along with classroom observation and class work, provide teachers with performance evidence which will show when a student has met proficiency expectations and identifies areas of weakness as well. The information gleaned from these assessments helps teachers identify remediation and enrichment opportunities for specific concepts and skills.

These **Investigations quizzes** for designated sessions in each unit serve as on-going formative assessment tools that teachers use to assess student progress throughout the unit of instruction. The data from these checks help teachers monitor student progress on a regular basis and provide information to help plan and differentiate instruction.

Investigations **observational assessment tools/checklists** are also used within their classroom as additional evidence of how students are progressing in relationship to the learning targets for the marking period.

Three **district-made benchmark assessments** are administered throughout the year to assess where students are in regards to the essential content addressed up to that point in time. (Kindergarten students are administered a mid and end-of-year benchmark.) In grades 1 to 5, these assessments consist of leveled questions for each of the 6 to 8 core standards plus open ended tasks. The results of these assessments are entered into Performance Tracker.

This **4- Sight assessment** is administered in January in grades 3 to 5 and data is used by the teachers to provide targeted review prior to the PSSA. Developed by Success for All Foundation with items field-tested in PA Schools, 4Sight provides diagnostic information on PA Standards and specific sub-skills to guide classroom instruction.

Data from the benchmarks assessment and 4-Sight is analyzed and used formatively by teachers to provide targeted supports to students. The information is also useful when refining the curriculum and designing professional development.

6. **If it becomes apparent through the data that students' conceptual understanding and math skills are not developing at either expected or previous rates, what steps will be taken?**
Data from the benchmarks assessment, PSSA and 4-Sight is analyzed and used formatively by teachers and math specialists to provide targeted supports to students. The information is also used by the math supervisor when refining the written district curriculum, aligning resources and designing professional development.

7. **What will parents see in an Investigations Classroom?**
In an Investigations classroom parents will see students working in a variety of groupings whether it be whole group, pairs, small groups or individually. Students will be communicating about the mathematics in a variety of ways. Sometimes the communication will be written, other times it will be oral, it may come in the form of building a model, or drawing a diagram or picture. Parents will see students investigating and exploring problems, comparing their results with others, discussing their strategies for solving the problems, explaining their answers, and considering their reasoning as well as the reasoning of their classmates. Parents will also see students using technology such as the calculator and computer and a variety of other tools to help deepen their understanding of mathematics.

8. What does an Investigations lesson look like?

Investigations lessons normally last about one hour. During the first part of the lesson called the introduction which lasts between 5-15 minutes, the teacher may pose a problem, explain the rules of a game, or read a story. The teacher is careful during the introduction not to "give away" the mathematics, so that students are given time to discover during the exploration part of the lesson.

The second part of the lesson called exploration is where the majority of the lesson is spent. During this time students will be actively engaged in solving problems, discussing strategies with their classmates, and recording their solutions in words, pictures, or numbers. In other sessions students may be playing mathematical games to practice skills or develop mathematical strategies. They may also be building geometric models or collecting data to organize and analyze. The teacher during this part of the lesson will be moving around the room among groups of students recording observations, interacting with students, asking probing questions to help him or her gain insight into how the student is processing mathematics. Questions such as "How did you get your answer?", "Can you describe your method and explain how it works?" "Can you find a second way to prove that?" are all questions that a teacher might use to require critical thinking of the student.

In the final part of the lesson called the summary, the teacher will pull the group back together as a whole class. This is the time students have the opportunity to reflect on the mathematics that they learned during the exploration part of the lesson. They may share approaches that they used to solve a problem. They may discuss the strategies they used when playing a game. They may relate the lesson of that day with the mathematics previously taught. Questions such as "Does your answer seem reasonable?" "What have you learned?" "What if you could only use…?" are all questions that a student could hear during the summary of the lesson.

9. What will homework look like?

Homework is an important means of communicating with parents. It can be a way to share math ideas that were presented that day, give a parent an opportunity to see how his or her child is working mathematically, and give the parent insight into how his or her child is learning and doing mathematics. In Investigations students may bring home only one or two problems and be asked to show how they solved the problem. They may be asked to show how they would solve it in a different way. Homework may also consist of game directions and a score sheet that the student will play with you or might be the collecting and recording of data in a way that makes sense to the student. Homework may also include a practice page or a review of the concepts presented in the ten-minute math activities.

10. How can I help my child with his or her homework?

When your child asks you for help try not to jump in with the answer to the problem. Rather ask questions to get your child started toward solving the problem. For example:
"How would you describe the problem in your own words?" "What have you come up with so far?" "Does this remind you of other problems that you have done?" "Where might you start this problem?" "Would drawing a picture or diagram help?" "What is the problem asking you to do?" Encourage your child to wrap his or her brain around the problem. Some of our best learning comes in the struggle. If your child becomes frustrated after having worked on a problem for a long time send a note to your child's teacher and communicate with them the difficulty your child had and ask the teacher to help your child or give you further suggestions on what you can do to help him or her.

If your child is working on a game, play the game with him or her. Have your child explain the rules to you and have him or her talk about the strategies that he or she chose to complete the game as well as the math. Games are an effective way to practice skills and develop efficient strategies.

Listen carefully to your child whether he or she is solving a problem, playing a game or collecting data. Through your observation you can gain great insight into the sense your child has about mathematics.

Display a positive attitude toward mathematics and the importance that it has in your child's daily life and future. Let him or her know that he or she can succeed.

To help with homework – which sometimes looks foreign to us – we can also:

- Be sure that we have read the background that the teacher has provided for us;
- See if we can see patterns in the work our children are bringing home;
- Ask probing questions of our kids when tackling homework:
    - What is the problem you're working on?
    - What did the teacher tell you to do? Or, what do the directions say?
    - What do you understand? What don't you understand?
    - Are there words or directions you are uncertain about?
    - Where do you think you should begin?
    - What do you already know that might help?
    - Can you draw a picture or make a table to work it out?
    - Let's talk through some steps, and see if we can't "talk it out."

Remember that it is o.k. to struggle with math problems! Through trial and error, children learn valuable skills.

If you are struggling with homework longer than is reasonable, simply jot a note to the teacher about the problem you had. He or she will address it the next day with the child, and even write you back if necessary.

We can also learn more about the *Investigations* program. We can attend informational meetings, and communicate with teachers when we don't understand something. In fact, dialogue is the most important tool we have!

11. How does Investigation meet the needs of all students?

Students in an Investigations classroom have an opportunity to work in heterogeneous groups. Students learn from one another not only through conversation, but through investigations that they complete collaboratively. By sharing their understandings with their classmates they have an opportunity to contribute to the learning of others. They also learn what it is like to work as a team to develop multiple solutions just as many people in the workforce do today.

Students who are learning English get the opportunity to connect the spoken and written language with numbers, symbols, drawings, pictures, demonstrations, and manipulatives. The manipulatives and models help the student make connections with what they are doing to solve a problem with what is being said about solving the problem.

The activities in Investigations are designed to bring out and build on mathematical thinking. Mathematics is communication. Some students when solving problems will come up with one solution. Others will come up with multiple solutions. It is important that all students explain their thinking, make sense of others' thinking, and consider the efficiency of various strategies that students offer. By doing problems differently and being able to explain their thinking, students have a deeper understanding of the mathematical concepts.

In addition to the Intervention and extension ideas in the teacher edition, there is also a separate Differentiation and Intervention Guide that provides checks along the way and ideas on how to remediate and enrich based on the results of those formative assessments. The curriculum map for each unit also lists supplemental activities, websites and resources to extend the ideas presented in the unit as well as provide extra support to those students when needed.

12. Why not just teach the standard algorithm right away?

If you teach the standard algorithm (or alternative algorithms for that matter) right away, students will often times only use that one method forgoing all others even when others are easier and faster. For instance, the equation that solves the problem, "There were 75 students on the playground. All 48 of the second graders left to go in. How many student are still on the playground?" could easily be solved using mental math by counting first to 50 from 48 and then jumping from 50 to 75, (which many students can do mentally because they are such friendly numbers) allowing the student to arrive at the solution of 27. Contrast that with the standard algorithm. First you line up the columns. Then you trade 1 ten from the 70 and make the 70 into 60 and the 5 into 15. Finally you subtract the ones (15 – 8 = 7) and then the tens (6 – 4 =2 or 60 – 40 = 20) arriving at the exact same place of a solution of 27.

This is not to say we do not ever want to develop these algorithms, they can be helpful, but if we teach the algorithms first it is very unlikely these students will ever develop the flexibility using number sense that we are looking for. Instead of first looking at the numbers and thinking what is the best way to do this arithmetic, they will often blindly follow the procedure of the algorithm. We see this in the way many of our older students and adults compute with whole numbers. Even when given a relatively simple equation to figure out mentally such as 102 – 97 many students will automatically write out the equation in columns and begin the standard algorithm or procedure. Teaching procedures before number sense is developed often limits their number sense and mental math flexibility in the long run.

13. What roles do traditional algorithms play in instruction?

The traditional paper and pencil algorithms have been part of the culture of mathematics for most every teacher today and for most every adult who has a child in the elementary or middle grades today. However, the question as to whether traditional algorithms should continue to play a role in the mathematics curriculum of the future, particularly at grades 1 through 7, has been of great interest in recent times. There are many schools of thought concerning algorithms, including replacing traditional algorithms with invented algorithms, teaching both traditional and invented algorithms, maintaining traditional algorithms as the hallmark of the curriculum, and replacing algorithms all together with calculators. The recent work relative to algorithm development and numerical power suggest the following:

- Students do not have to learn traditional paper and pencil algorithms to be numerically powerful.
- Students can be introduced to traditional algorithms, but traditional algorithms need to be developed differently than they have in the past.
- All students can and should do more calculations mentally.

Numerically powerful children are computationally competent – they know when it makes sense to use a calculator and when it makes sense not to, and they can accurately and efficiently complete numerical calculations mentally or with paper and pencil, using procedures that make sense to them. The evidence is growing that numerical power, including accurate and efficient computational competence, can be achieved without exposure to traditional paper and pencil algorithms (see e.g., Wood & Sellers, 1997). In particular, instructional programs can be designed to enable children to invent algorithms that evolve naturally from the child's understanding of number, numeration, and basic facts. Furthermore, meaningful invented algorithms can produce the computational competence needed for success on achievement tests, for access to and success with further mathematics, and most importantly, for mathematical literacy in an ever-increasing mathematical world.

If numerical power can be developed without introducing traditional algorithms, does it follow that traditional algorithms be omitted from the curriculum? The answer, of course, is not simple. It was not an accident that traditional algorithms became "traditional." When calculators did not exist, it was imperative that paper and pencil procedures be taught to enable one to complete calculations accurately with any size numbers. Traditional algorithms attained their status because of their efficiency in completing large-number calculations. Thus, if the only goal relative to computational competence is to develop efficient ways to complete calculations, then traditional algorithms are probably the best means to that goal. Because our goals have expanded to the development of numerical power, however, a concern for efficiency alone is not appropriate. For example, in a highly publicized item of the Third National Assessment of Educational Progress (NAEP, 19xx) exam, more than half of the 13-year-olds responded that 7/8 + 12/13 was about 19 or 21, thus indicating very little sense of the reasonableness of the answer to a calculation. Knowing a traditional algorithm for finding an exact answer isn't very useful in the real world if one's number sense relative to the calculation and the answer is poor.

When the majority of a child's time in mathematics is spent memorizing what the child considers to be nonsense, he or she soon abandons altogether his or her efforts to make sense of mathematics.

For many years, traditional instruction has attempted to develop algorithms with understanding. Unfortunately, the traditional approach does not help most children make sense of these algorithms, and the students resort to rote memorization. Furthermore, the work of Kamii, Lewis, and Livingston (1993) suggests that the rote learning of traditional paper and pencil algorithms actually can interfere with the child's development of number sense. When the majority of a child's time in mathematics is spent memorizing what the child considers to be nonsense, he or she soon abandons altogether his or her efforts to make sense of mathematics. Furthermore, when children believe that facility with traditional paper and pencil algorithms is the most important goal for elementary school mathematics, many children stop relying on their natural abilities to think and reason about numbers and immediately apply these algorithms, regardless of the kinds of numbers involved in a calculation.

More students can do more reasonable-size number calculations mentally than has been believed or expected in the past. Therefore, instruction should be organized to promote the development of these abilities. Instructional programs need to provide regular mental calculation experiences, allowing children to share their thinking and reasoning. Classroom conversations about mental calculation strategies help children develop a sense of the kinds of actions on numbers that lead to simpler and equivalent numerical expressions. Numerical power is promoted through frequent opportunities with mental calculations, and as students become numerically powerful, they do more calculations mentally.

Summary: Traditional paper and pencil computational algorithms must be reconceptualized as part of a total rethinking of the curriculum aimed at developing numerical power. Developing traditional algorithms with understanding will not happen by spending more time doing what we have always done. Although traditional algorithms are not necessary, they can be introduced as part of a program aimed at developing numerical power. If traditional algorithms are introduced, however, their introduction should be preceded with extended number sense development, and they should be presented as just another way of doing calculations. Most important, the meaningful development of any computational algorithm seems possible only when that algorithm evolves naturally from one's understandings of numbers, number relationships, and operation meanings.

14. Why have Flexible methods for computation?

"As students move from third grade to fifth grade, they should consolidate and practice a small number of computational algorithms for addition, subtraction, multiplication, and division that they understand and can use routinely…Having access to more than one method for each operation allows students to choose an approach that works and best fits the numbers of a particular problem." (NCTM)

Benefits of allowing flexible methods

- Fewer errors
- Students develop number sense
- Match strategies utilized for mental math and estimation – do not need to be taught as separate skills
- Often are faster and easier if student has good number relation (basic fact) skills
- Basic fact and ability to jump from number to number mentally is fostered because it is so valuable in mental math strategies
- Students do what makes sense and comes naturally to them.

15. What about computation and basic facts?

We still want students to know their basic facts and how to compute, but we also want our students to be able to reason, problem solve, and communicate mathematics. American students often rank near the bottom in international testing in the areas of higher order thinking skills such as reasoning, conceptualization, and problem solving. Employers today are looking for employees that can look at a problem and think of possible ways to solve it. They are looking for employees that have good number sense, skills in spatial visualization, competence in using and interpreting data, and familiarity with technology.

In Investigations a central objective is for students to learn about numbers, their relationships and operations. A great deal of time is spent on number and number sense with the focus being the development of students' own strategies for solving problems.

In addition to fact practice being part of daily routines and workshop time in Investigations, the program, First in Math, is being used in grades 1 to 5 as an additional resource to help students learn their math facts while developing problem solving skills.